## Professor Mordecai Kurz, September 2009

## Risk and Insurance: Two Problems For Study

## 1. Risk and Optimal Crop

A farmer believes there is a $50 \%-50 \%$ chance the next crop season will be abnormally rainy. Assuming his utility function is $\log ($ i.e. $\mathbf{u}(\mathbf{Y})=\log \mathbf{Y}$ with $\log$ to base e) his expected utility has thus the form

$$
E u=\frac{1}{2} \log \left(Y_{N R}\right)+\frac{1}{2} \log \left(Y_{R}\right),
$$

where $\mathrm{Y}_{\mathrm{NR}}$ and $\mathrm{Y}_{\mathrm{NR}}$ are the farmer's incomes in the two states of "normal rain" and "rainy" season, respectively.
(a) Suppose the farmer must choose between two crops that promise the following incomes:

| $\underline{\text { Crop }}$ | $\underline{\mathbf{Y}_{\mathbf{N R}}}$ | $\mathbf{Y}_{\mathbf{R}}$ |
| :--- | :--- | :--- |
| Wheat | $\$ 28,000$ | $\$ 10,000$ |
| Corn | $\$ 19,000$ | $\$ 15,000$ |

Which crop will the farmer plant?
(b) Suppose the farmer can plant half his field with each crop. Would he choose to do so? Why?
(c) If the farmer chooses to mix wheat and corn in (b), is there an optimal mix that maximizes his expected utility?
(d) Would wheat crop insurance, available to farmers who grow only wheat, which costs $\$ 4,000$ and pays $\$ 8,000$ in the event of a rainy growing season, cause this farmer to change what he plants?

## 2. Using Options to Speculate and Hedge

Consider the following table of option prices of eBay Company at some date when the stock was selling for $\$ 86.96$ All options expire one month later.

| Strike Price | market price of call | market price of put |
| :---: | :---: | :---: |
| 80 | 8.05 | 0.30 |
| 85 | 3.85 | 1.05 |
| 90 | 1.25 | 3.50 |

(A) The stock has been extremely volatile and you do not expect the price to stay in this range over the next month but you are not sure which way it will go. Design a strategy that will benefit you if the price will not be in the $\$ 80-\$ 90$ range a month later.
(i) Draw a graph of the value of your option position as a function of the expiration price of the stock
(ii) How much does the position cost you?
(ii) Draw the profit from your position.
(B) You own a 100 shares of eBay and you are worried about the stock dropping below $\$ 80$. Design a strategy which will protect you against the stock dropping below $\$ 80$.
(i) Draw a graph of the value of your option position as a function of the expiration price of the stock
(ii) How much does the position cost you?
(C) From the information provided in the table deduce the annual interest rate in the market (i.e. the interest rate paid for one year loans) on the date at which these prices were posted?

